Waveguide Input Method for Time-Domain Wave Propagation Algorithms

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***Abstract*—A waveguide feed method is constructed for time-domain wave propagation algorithms. The concept is to connect a region of homogeneous index of refraction to the waveguide region by a refractive index taper. The first example of this method finds the coupling efficiency for a given input into a waveguide of a particular index geometry. The second example of this method reveals the evolution of the propagation constant through a horn geometry.**

I. Introduction

MANY methods have been proposed for time domain simulation of optical waveguides. They are all based upon a self-consistent formulation based upon Maxwell's equations. The most common method has been Finite Difference Time Domain (FDTD) [1]. Its simplicity of implementation and its general nature make this method popular. Still another method is the Time-Domain Beam Propagation Method demonstrated by Gomelsky and Liu [2]. This method is capable of handling dispersive media. Finally, there is the Wave Propagation Method (WPM) proposed by Chan and Liu [3] which is a numerically efficient time-domain method. All of the above methods fall under the category of Initial Value Problems (IVP).

II. Specification of Initial Value Problem

Generally, in a waveguide problem, one specifies the frequency of the optical field or, equivalently, its wavelength in free space. Then, one finds the corresponding propagation constant, β, in the given structure through any wave propagation method such as those mentioned above. However, an IVP formulated along the time axis requires an input at one point in time specified for all space. Since β in the waveguide for the given optical frequency is initially not known, it is not possible to correctly describe the spatial distribution of the field in the waveguide region even at the initial point of time. Specifying an initial spatial field distribution in the waveguide arbitrarily is equivalent to guessing β. A time-domain wave propagation algorithm with this input will evolve in time according to an optical frequency corresponding to this guessed β dictated by the dispersion relation of the waveguide. In general, this frequency is not the one that was specified for the problem and thus, the IVP is not well-posed.

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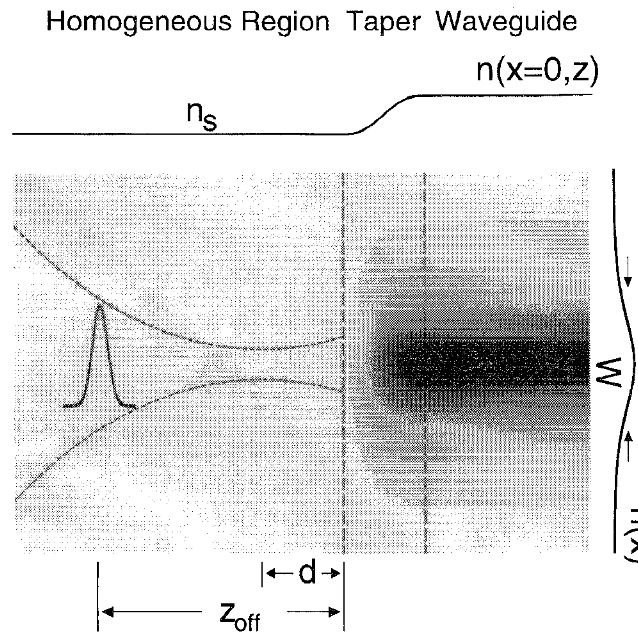

Fig. 1. Input geometry for the Gaussian pulse launch.

On the other hand, in a region of homogeneous permittivity, analytical solutions of Maxwell's equations exist and consequently, the spatial field distribution for any frequency can be exactly specified. Therefore, if the input field exists initially only in a homogeneous region, then the IVP can be posed correctly. For propagation in a waveguide, one can add a homogeneous region along the propagation direction to correctly pose the IVP. This homogeneous region may be connected with the waveguide region by means of an index transition. The index of refraction will then gradually change from that in the bulk to that of the waveguide over some longitudinal distance. The region of index change should follow a smooth function to ensure a smooth transition. This concept is shown pictorially in Fig. 1.

III. Numerical Demonstration

To demonstrate this concept, consider the problem of time-domain pulse propagation in a two-dimensional waveguide. The input for this IVP is a snapshot of the pulse at $t = 0$. In a region of homogeneous permittivity, one solution to Maxwell's equations is that of a Gaussian beam and the propagation of the pulse in space follows the Gaussian beam trace. Therefore, the input field distribution is a spatial pulse modulation over the Gaussian trace. Our two-dimensional spatial input is

$$E(x,z) = \left\{ \sqrt{\frac{w_{ox}}{w_x(z)}} \exp\left[-\frac{x^2}{w_x^2(z)}\right] \cdot \exp\left[i\{kz - \eta(z)\}\right] \cdot \exp\left[i\frac{kx^2}{2R_x(z)}\right]\right\} \cdot \exp\left[-2\ln 2\left(\frac{z - z_{\text{off}}}{c_s \Delta t_0}\right)^2\right]$$

This field has two parts: an elliptic Gaussian trace and a pulse modulation. For the Gaussian, the parameter $w_x^2(z) = w_{ox}^2[1+(z+d)/z_{ox}]$ while the parameter $R_x(z) = (z+d)\{1+[z_{ox}/(z+d)]^2\}$ and $\eta(z) = 1/2 \tan^{-1}[(z+d)/z_{ox}]$. The Rayleigh range is $z_{ox} = w_{ox}^2 k/2$ where w_{ox} is the spot size. The propagation constant k is defined as $k = 2\pi n_s/\lambda_o$ where n_s is the substrate index and λ_o is the optical wavelength in free space. The parameter d is the distance from the edge of the taper to the focal point of the Gaussian. For example, $d = 50$ μm means that the Gaussian beam waist (the focal point) is 50 μm from the start of the taper measured back into the bulk region. The second part of the field is the modulation. The elliptic Gaussian can be viewed as a static field pattern over which a modulation, in our case an optical pulse, may pass. The pulse is centered at z_{off} and $c_s = c_o/n_s$ where c_o is the speed of light in vacuum. The full-width at half-maximum pulsewidth in time, Δt_0, is thus converted to a corresponding width in space.

One application of this input method is to determine the launch efficiency for a given input pulse into a longitudinally invariant waveguide. The launch efficiency varies with w_{o_x} and d. The input energy is found from a 2-D integration of the electric field intensity in the bulk region. Then, when the pulse has fully entered the waveguide and the radiation fields have subsided, one may similarly again determine the energy of the pulse. The ratio of these two energies is the launch efficiency.

The WPM method was chosen for the simulation because of its efficiency of memory and computation. A substrate of $LiNbO_3$ was selected at an optical wavelength of 1.3 μm. Then, the refractive index was found from the relation: $n^2(x) = n_s^2 + 2\Delta n \operatorname{sech}^2(2x/W)$ where $n_s = 2.14532$ and $\Delta n = 0.02$ while the width parameter, W, was chosen to be 2 μm for a single mode to exist. The taper was found from a cubic spline with natural boundary conditions connecting the substrate index to the waveguide index over a distance of 50 μm. An ultrashort pulse with a pulsewidth $\Delta t_0 = 500$ fsec was used. Consideration for the efficiency and accuracy of the mapping algorithm determines a definite link between the temporal and the longitudinal spatial stepsize, Δt and Δz [3]. The spatial stepsize is chosen to be $\Delta z = c/n_s \Delta t$ for point to point mapping in the substrate. The values of the computational stepsizes were $\Delta x = 0.25$ μm, $\Delta z = 1$ μm and $\Delta t = 7.15114$ fsec.

Fig. 2 shows a contour plot of the efficiency expressed in percentage. The data is taken just as the pulse has completely passed a distance of 600 μm from the start of the taper. The first observation is that the efficiency varies as the longitudinal position of the focal point, d. The peak efficiency appears when the focal point lies just within the taper region. The efficiency

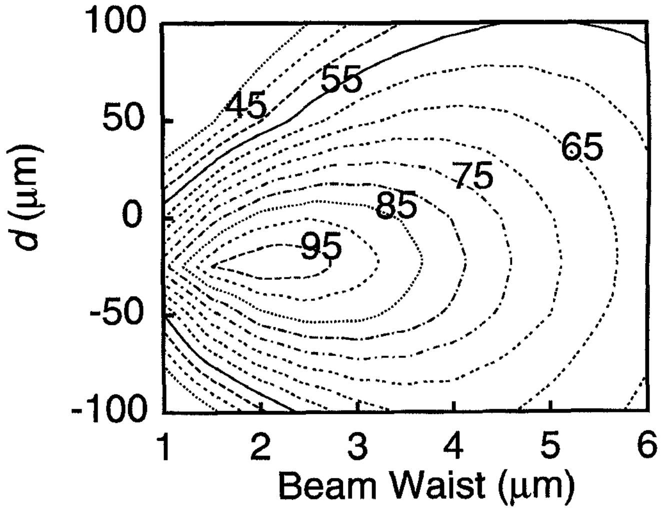

Fig. 2. Launch efficiency as a function of the beam waist, w_{o_x}, and the Gaussian offset, d. The Gaussian offset distance is positive into the homogeneous region and negative into the taper. The contour values are expressed in percentage.

drops faster when the focal point is located behind the taper than when the beam is placed within the taper. The reason is that in a homogeneous region, the field diffracts according to Maxwell's equations, thus spreading energy well away from the waveguide core, while in the taper, the field is quickly guided and more energy is captured. The maximum efficiency, 99.68%, was observed at a beam waist of 2 μm and $d = -25$ μm into the taper. Another observation is that the efficiency is a function of the focusing. Clearly, the best coupling occurs when the Gaussian has a beam waist closely matching the width parameter, W, of the sech^2 waveguide, as expected.

The computational size of the domain was 200 μm in the transverse direction and 400 μm in the longitudinal direction. This amounts to 9.7 Mbytes of storage. The transverse direction can be decreased if the absorber at the transverse boundaries is replaced with some absorbing boundary condition. For this run, 1000 timesteps were used to fully ensure that the computational window passed a detector located 600 μm from the start of the taper. Also, the initial location, z_{off}, of the pulse is 200 μm behind the taper interface which puts the pulse in the center of the initial computational window.

The coupling efficiency could be increased if the index of the homogeneous region were chosen to be equal to the peak index, $n_{\max} = n(x = 0)$, of the waveguide. In this way, the index near the peak of the pulse would not change from the bulk region to the waveguide. This would maximize the coupling process. While this is true for other wave propagation methods such as FDTD, it is not true for WPM. In WPM, the field is initially mapped from point to point in the homogeneous region. If the peak index is chosen for the bulk region, then one should adjust the spatial stepsize as $\Delta z = c/n_{\max}\Delta t$ to achieve the same benefit as before. The central portion of the pulse will undergo the least amount of distortion. However, when the pulse reaches the waveguide, there is interpolation error for the field outside of the waveguide, namely the field of the substrate. Since the substrate occupies the majority of the window, this degradation should be avoided. Therefore, the choice of $\Delta z = c/n_{\max}\Delta t$ is not better than that of $\Delta z = c/n_s\Delta t$ for WPM.

As a second application, this input method is used to calculate the propagation constant, β, as a 500–fsec pulse undergoes a horn transition. The horn is formed from the same sech^2 profile as in the above example. The width parameter is ramped via a polynomial spline in the horn region.

To extract β, a spatial method is used. Since the phase of the electric field for a single mode guide varies as $\exp(i\beta z)$, one can deduce β directly from the instantaneous phase of the electric field at the peak of the transverse profile. Explicitly, the phase is found from the ATAN2 function in FORTRAN. Then, the slope of the phase with respect to distance yields β. To average over any radiation effects, one can use a longer range. A 2π range centered about the peak of the pulse was chosen. A more precise determination of β comes from measuring the parameter λ_g defined as $\lambda_g = 2\pi/(\beta - k_s)$. The analytical result at $W = 2$ μm is $\lambda_g^{\text{ana}} = 130.20$ μm. As W increases, the guide becomes multimode. However, the input Gaussian will only excite the lowest order mode due to even symmetry. Note that for $W = 3$ μm, $\lambda_g^{\text{ana}} = 103.95$ μm and at $W = 4$ μm, $\lambda_g^{\text{ana}} = 92.66$ μm.

The evolution of λ_g through a horn transition is plotted in Fig. 3. The peak efficiency parameters given above were chosen to obtain a steady field. Then, a horn was introduced at $z = 700$ μm. The top three curves represent a transition of waveguide width from $W = 2$ μm to $W = 3$ μm horn lengths of 200 μm, 500 μm, and 1000 μm. The final value of λ_g was 103.64 μm. This shows that even short horn transitions can settle quickly. The bottom curve represents a transition from $W = 2$ μm to $W = 4$ μm. The final value of λ_g was 92.39 μm. Therefore, the WPM program correctly predicts the evolution of the pulse as it passes through a horn.

IV. Conclusion

In conclusion, a feed mechanism utilizing a longitudinally tapered index profile is presented for time-domain wave propagation methods. This feed mechanism overcomes the launch problem faced by IVP's. This input method is demonstrated for the calculation of the coupling efficiency of a short optical pulse launched into the tapered waveguide and the quick extraction of the propagation constant from spatial phase variations.

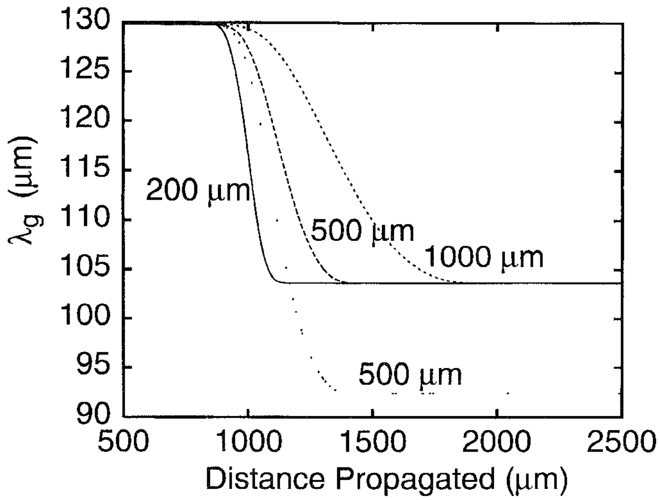

Fig. 3. Evolution of λ_g through different horn geometries. The top three curves represent a transition from $W = 2$ μm to $W = 3$ μm for horn lengths of 200 μm, 500 μm, and 1000 μm. The bottom curve represents a transition of $W = 2$ μm to $W = 4$ μm for a horn length of 500 μm.

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